Retirement Portfolio and Variable Annuity with Guaranteed Minimum Withdrawal Benefit (VA+GMWB)

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About Ibbotson Associates

Ibbotson Associates opened its doors in 1977 to bridge the gap between modern financial theory and real-world investment practice. Professor Roger G. Ibbotson, the company founder, pioneered the collection of the requisite historical data used in asset allocation and quantified the benefits of diversification. Ibbotson continues to provide solutions to investment and finance problems for a diverse set of markets.

Entrusted to create asset allocation models for many of the largest companies in the finance and investment industries, Ibbotson Associates is a leading provider of retirement advice programs and investment consulting services to institutions. Ibbotson Associates is a registered investment advisor and wholly owned subsidiary of Morningstar, Inc.

Executive Summary

The total assets under management (AUM) for variable annuity accounts at the end of 2006 reached $1.36 trillion, an increase of 38.2% since the end of 2001. The dominant sales driver for variable annuities in recent years was the guaranteed minimum withdrawal benefit (GMWB). Part of the reason is that the shift from Defined Benefit (DB) plans to Defined Contribution (DC) plans has created a void for retirees – the loss of guaranteed income in retirement, therefore, retirees need to find other guaranteed income sources. In this paper, we introduce a basic variable annuity (VA) product with a guaranteed minimum withdrawal benefit (GMWB), and use empirical analysis and Monte Carlo simulations to study the benefits and costs of including VA with GMWB in a retirement income portfolio. This continues our studies on incorporating insurance products into an investor's overall portfolio (e.g., Chen and Milevsky (2003), Chen, Ibbotson, Milevsky, and Zhu (2006 & 2007)). [1]

We have developed a hypothesis that the GMWB will help improve the overall retirement income levels without increasing income risk levels. We employed the income risk or income semi-deviation, which is defined as the standard deviation on negative income changes over the last period, for a series of simulation analyses across three scenarios: 1) a diversified asset allocation VA account with GMWB; 2) a diversified traditional non-annuity portfolio (such as mutual funds); and 3) a combination of VA+GMWB products and non-annuity products in a portfolio context. In the combined portfolio, a portion of the fixed income /cash allocation is replaced with a more aggressively allocated VA, which will leave the remaining mutual fund portfolio with a higher equity allocation than the original mutual fund portfolio. The analysis compared the traditional mutual fund portfolio with the combined portfolio assuming a fixed percentage withdrawal rate (5%) on the non-VA portion of the portfolios, and a fixed 5% on the benefit base of the VA portion.

Both empirical results and Monte Carlo simulations show that the combined portfolios have lower average negative income return, lower semi-deviation, higher average income return, and higher total income withdrawals. (See VI Appendix C for Glossary.)

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[1] The research was conducted by Ibbotson and sponsored by Nationwide Financial®. Special thanks go to Nationwide Financial, specifically, John M. Kawauchi, Harold C. Schafer and Antonio E. Morello, CFA for helpful discussions on this white paper, especially on applying the concept of semi-deviation on income amount as a measurement of income risk and developing the hypothesis that the GMWB will help improve the overall retirement income levels. Ibbotson would also like to thank Roger Ibbotson, Frank O'Connor, and Moshe Milevsky for valuable comments.

In general, as we increase the equity exposure of the traditional non-annuity investments, the portfolio provides higher total income with higher semi-deviation (thus higher income risk). The addition of VA+GMWB to the retirement portfolios (replacing cash or fixed income allocations) increases total income while it decreases income risk. That is, the guaranteed VA portion has no income risk so it helps to lower the income risk for the overall combined portfolios, and the combined portfolios have a higher equity allocation which contributes to an increase in total income.

We then analyzed the shortfall income risk and average or median sustainable income level for the three scenarios over a 30-year horizon. We found that all combined portfolios have higher median income levels and lower shortfall income risk than stand-alone traditional mutual fund portfolios. Therefore, for a 30-year horizon, adding VA+GWMB to the conservative, moderate conservative, and moderate portfolios is beneficial in that it increases the average sustainable income and decreases the shortfall income risk.

Although there is potential risk that insurance companies could default on these VA guarantees, we did not specifically incorporate default risks on these VA contracts into our analysis, since almost all VA contracts are offered by insurance companies with very high credit standings. We believe the default risk would lower the amount of the VA+GMWB benefit in a retirement portfolio, but it would not have necessarily changed the main analytical results.

I. Introduction

The growing number of retiring baby boomers, the paradigm shift away from defined benefit pensions to defined contribution pensions, medical advancements, longer life expectancies, and increased uncertainty surrounding Social Security benefits dramatically increase the burden of retirees to finance retirement spending. Many retirees, who rely on their own personal savings in retirement, not only face market risk, but also retirement income risk. Today, most retirement assets are invested in traditional mutual fund portfolios, and they don't offer effective protections against market downturn or retirement income risk. There are some instruments, for example variable annuities and payout annuities, that can help investors hedge market risk and retirement income risk. In this paper, we concentrate our effort on analyzing variable annuities with lifetime guaranteed withdrawal benefit (GMWB for life).

According to the latest issue of *Annuity Market News*, total assets under management (AUM) for variable annuity accounts at the end of 2006 reached $1.36 trillion, an increase of 38.2% since the end of 2001. The dominant sales driver for variable annuities in recent years was the guaranteed minimum withdrawal benefit (GMWB). In this paper, we study the role of variable annuities with a lifetime guaranteed minimum withdrawal benefit (GMWB) in managing market and retirement income risk, and more specifically, we use empirical analysis and Monte Carlo simulations to study the benefits and costs of including VA with lifetime GMWB in a retirement income portfolio.[2]

Our hypothesis is that GMWB in VA products can potentially improve the overall retirement income levels without increasing income risk levels. We developed the analytical framework regarding the measurement of income risk, the modeling of portfolios, and the graphical presentation of the results. We introduced the income risk-- the semi-deviation on income changes or income returns over the last period.

We ran a series of simulation analyses across three investment scenarios: 1) stand-alone VA products with these guarantees; 2) stand-alone traditional non-annuity products (such as mutual funds); and 3) a combination of VA products and non-annuity products in a portfolio context. The income return is defined as the income change in percentages in two consecutive years. In the combined portfolio, a portion of the fixed income /cash is replaced with a more

[2] We assume that there is no default risk in VA+GMWB contracts, and the fees do not change in the future. We also do not consider the scenario that an investor might need to withdraw the VA investment in a lump sum, which could incur surrender charges and loss of the guaranteed withdrawal benefits.

aggressively allocated VA, which will leave the remaining mutual fund portfolio with a higher equity allocation than the original mutual fund portfolio. The analysis compared the traditional mutual fund portfolio with the combined portfolio assuming a fixed percentage withdrawal rate (5%) on the non-VA portion of the portfolios. Both empirical results and Monte Carlo simulations showed that the combined portfolios had lower average negative income return and semi-deviation, and higher average total income return and total income withdrawals. (See VI Appendix C for Glossary.)

In the other framework, to effectively evaluate the risk-return tradeoff of different retirement income patterns, we examined the average sustainable income level versus the shortfall income risk relationship for the above-mentioned three scenarios. The shortfall income risk is defined as the risk of running out of income when the market has performed poorly for an extended period of time. We believe that this framework is also more important in examining the "portfolio risk-return" in retirement income setting than the traditional mean-variance framework. We found that all studied combined portfolios have higher average income levels and lower income shortfall risks than stand-alone traditional mutual fund portfolios. We see this framework as an extension of the work by Chen and Milevsky (2003). That is, to continue exploring more efficient ways to help investors build retirement income portfolios that maximize average sustainable income levels while minimizing shortfall income risk.

The rest of the paper is organized as follows. Section II introduces VAs with lifetime GMWB. Section III introduces the hypothesis. Section IV presents empirical analysis and Monte Carlo simulation results. The conclusions are given in section V. Glossary is presented in section VI Appendix C.

II. What is VA + Lifetime GMWB?

A recent innovation in the variable annuity products is the guaranteed minimum withdrawal benefit (GMWB) rider on variable annuities (VA+GMWB). The GMWB is often referred to as a benefit rider. This put-option-like rider can be purchased for a fixed term (e.g. 20 years), or for life. We will focus on the GMWB rider for life in this study. The GMWB rider for life gives annuitants the ability to protect their retirement investments against downside market risk by allowing the annuitant the right to withdraw a fixed percentage (e.g. 5%) of the benefits base each year until death. The benefits base can step up and will be reset to the high-water mark of the contract value on the rider anniversary date when market has performed well. The remaining contract value at death will be paid to beneficiaries, which removes the investor concern about giving up liquidity to the heirs.

With the VA+GMWB products, automatic annual resets are available after the contract is purchased. The best aspect of this guarantee is that it protects annuitants against any nominal investment losses that would have been incurred without losing the benefit of upside gain. In exchange for this benefit, the annuitant pays a fee each year. For example, suppose that John's initial investment was $1,000,000. Due to downturns in the economy, the investment suffers a 20% loss the next year and the contract value decreases to $800,000. Since John had purchased a guaranteed minimum withdrawal benefit with a rate of 5%, he will still be able to withdraw $50,000 each year no matter if the loss were 20% or 50%. However, if the net return is a 20% gain instead of a 20% loss, the contract value would be $1,200,000. The benefit base will be reset to $1,200,000 on the next anniversary day so that John is able to withdraw at least $60,000 each year thereafter.

In summary, some typical features of the GMWB rider for life include:

- A guaranteed lifetime withdrawal rate (e.g. 5%) on the benefit base. The benefit base is the high-water mark of the contract value on the rider anniversary date.
- A step-up feature allowing the investor to lock in a higher amount for the benefit base guarantee on the rider anniversary date.
- An Annual rider fee, which typically ranges from 0.35% to 0.75% of the benefits base.
- A payout of remaining contract value to beneficiaries at death.

Table 1 illustrates characteristics of the VA+GMWB, assuming that one purchased the contract at the beginning of 1979. Note that the benefits base is the high-water mark of the contract value, i.e. it steps up whenever the contract value exceeds the previous year's benefit base. Therefore, the guaranteed withdrawal or income never decreases. By 2006, the income would have reached $191,578, mainly due to the strong market performance over 28 years (1979—2006). The benefit base and the remaining contract value at the end of 2006 would have been $3,831,558 and $2,651,806, respectively.

Table 1. Conceptual Illustrations of VA+GMWB

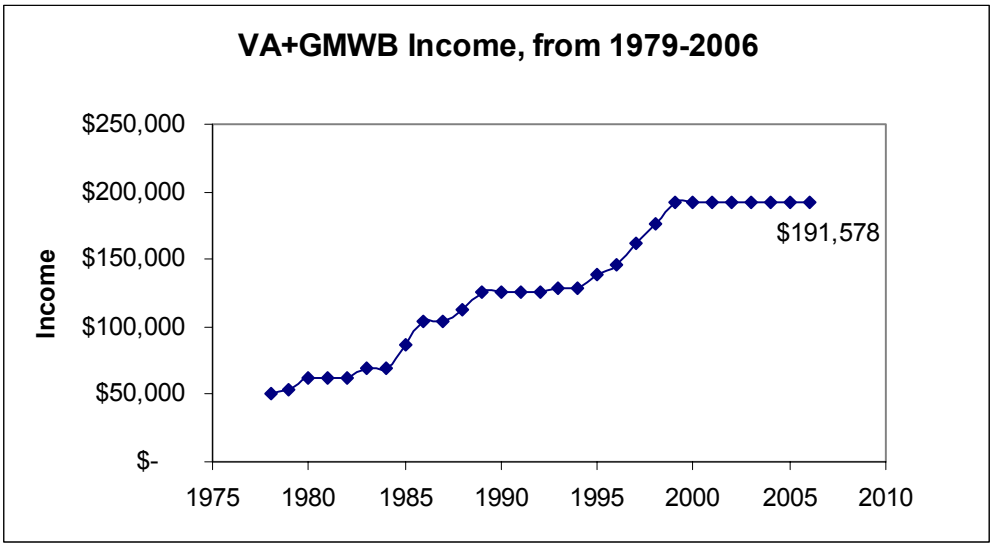
	VA+GMWB Net Return	Contract Value	Benefit Base	Guaranteed Income	Rider Fee
		$ 1,000,000	$ 1,000,000	$ (50,000)	$ (6,000)
1979	14.63%	$ 1,082,126	$ 1,082,126	$ (54,106)	$ (6,493)
1980	22.47%	$ 1,251,023	$ 1,251,023	$ (62,551)	$ (7,506)
1981	-2.46%	$ 1,151,956	$ 1,251,023	$ (62,551)	$ (7,506)
1982	15.30%	$ 1,247,439	$ 1,251,023	$ (62,551)	$ (7,506)
1983	18.23%	$ 1,392,037	$ 1,392,037	$ (69,602)	$ (8,352)
1984	4.06%	$ 1,367,388	$ 1,392,037	$ (69,602)	$ (8,352)
1985	33.52%	$ 1,721,640	$ 1,721,640	$ (86,082)	$ (10,330)
1986	27.22%	$ 2,067,582	$ 2,067,582	$ (103,379)	$ (12,405)
1987	5.12%	$ 2,051,779	$ 2,067,582	$ (103,379)	$ (12,405)
1988	16.49%	$ 2,255,230	$ 2,255,230	$ (112,761)	$ (13,531)
1989	18.46%	$ 2,522,024	$ 2,522,024	$ (126,101)	$ (15,132)
1990	-10.53%	$ 2,130,189	$ 2,522,024	$ (126,101)	$ (15,132)
1991	23.87%	$ 2,463,729	$ 2,522,024	$ (126,101)	$ (15,132)
1992	2.63%	$ 2,383,461	$ 2,522,024	$ (126,101)	$ (15,132)
1993	14.30%	$ 2,562,788	$ 2,562,788	$ (128,139)	$ (15,377)
1994	-1.13%	$ 2,391,922	$ 2,562,788	$ (128,139)	$ (15,377)
1995	23.71%	$ 2,781,570	$ 2,781,570	$ (139,078)	$ (16,689)
1996	11.50%	$ 2,927,717	$ 2,927,717	$ (146,386)	$ (17,566)
1997	16.89%	$ 3,230,578	$ 3,230,578	$ (161,529)	$ (19,383)
1998	15.21%	$ 3,513,505	$ 3,513,505	$ (175,675)	$ (21,081)
1999	15.52%	$ 3,831,558	$ 3,831,558	$ (191,578)	$ (22,989)
2000	-5.34%	$ 3,423,861	$ 3,831,558	$ (191,578)	$ (22,989)
2001	-11.06%	$ 2,854,250	$ 3,831,558	$ (191,578)	$ (22,989)
2002	-15.40%	$ 2,233,264	$ 3,831,558	$ (191,578)	$ (22,989)
2003	27.01%	$ 2,563,937	$ 3,831,558	$ (191,578)	$ (22,989)
2004	11.46%	$ 2,618,572	$ 3,831,558	$ (191,578)	$ (22,989)
2005	5.86%	$ 2,544,903	$ 3,831,558	$ (191,578)	$ (22,989)
2006	13.80%	$ 2,651,806	$ 3,831,558	$ (191,578)	$ (22,989)
			Total:	$ (3,702,540)	$ (444,305)

* The net returns are calculated based on a moderate aggressive portfolio (see table 2A) with the assumed fee structure shown in table 2D.

** The withdrawal is assumed at the beginning of each year.

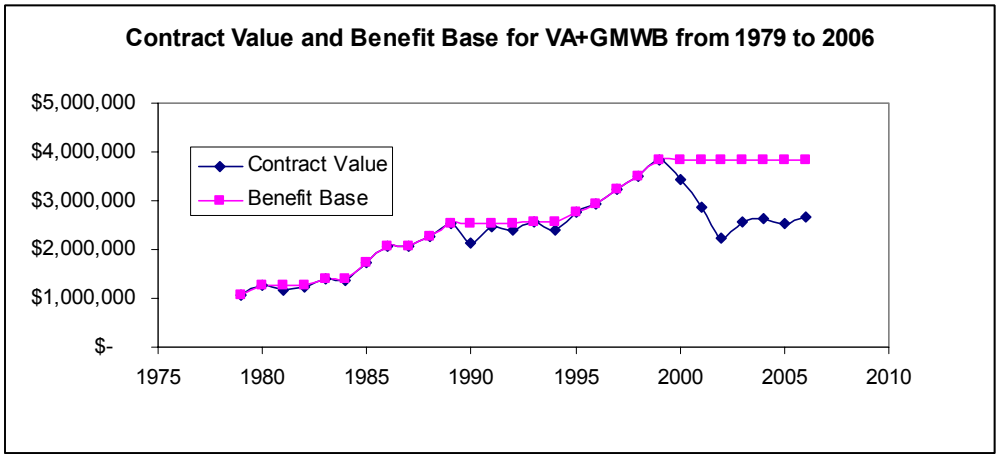
Chart 1A plots the guaranteed income levels shown in table 1. It is noted that the nominal income level steps up and never decreases, and reaches $191,578 by 2006. The first payment of $50,000 starts at the beginning of 1979. The stock market crash from 2000 did not reduce the income, and the guarantee keeps the income level flat. The VA+GMWB contract value and benefit base are plotted in Chart 1B. Note the contract value in 2006 is still below the guaranteed base, therefore, the income level would remain flat in the future till the contract value exceeds the benefit base.

Chart 1A. Guaranteed Withdrawal from 1979 to 2006 for VA+GMWB



* The assumed fee structure is shown in table 2D.

Chart 1B. Contract Value and Benefit Base for VA+GMWB from 1979 to 2006



* The assumed fee structure is shown in table 2D.

III. Hypothesis of Adding VA+GMWB to a Retirement Portfolio

As we know, traditional mutual fund portfolios may suffer from market downside risk and jeopardize retirement income. Our hypothesis is that adding VA+GMWB to traditional portfolios will improve the overall retirement income levels without increasing income risk levels.

The income risk describes the downside volatility of the change in income from year to year. Furthermore, as the allocation to VA+GMWB increases, the income risk decreases. A VA+GMWB by itself carries no income risk due to the high-water-mark feature, which allows the income to only go up and never go down.

The idea is to replace some fixed income or cash allocation in a traditional portfolio with VA+GMWB, resulting in a slightly more aggressive combined portfolio. We will show that the VA+GMWB has no income risk which helps to lower the income risk for the combined portfolios, and the combined portfolios have higher equity weight which contributes to increase total retirement income over time. [3]

[3] In this paper, we presented results in nominal terms. We did not explicitly consider inflation. A real (as opposed to nominal) analysis will result in guaranteed payments possibly falling over time for VA+GMWB, which creates a higher downside volatility or semi-deviation.

IV. Empirical Analysis

Monte Carlo Simulations

We employ Monte Carlo simulation to study the dynamics of VA+GMWB and traditional mutual fund portfolios over various market return scenarios. With Monte Carlo simulations, one can specify a probability distribution for variables such as market returns, volatilities and covariance. In one simulation path, a path of random market returns for each asset class is generated. The portfolio value is calculated by following equation (1) or (2) in Appendix A. Each path represents one possible investment horizon experience. A large number of paths (5,000) are tabulated to determine the outcomes over broad spectrum of risk and return scenarios. Based on the outcomes, we can analyze the risk information, impact of different asset allocations, and the probability of reaching retirement goals, etc. Therefore, Monte Carlo simulation is generally considered a better tool to provide comprehensive analysis than the deterministic method. Monte Carlo simulation allows us to view projections of best- and worst-case scenarios.

Model Portfolios and Capital Market Returns

Four diversified asset allocation portfolios—conservative, moderate conservative, moderate, and moderate aggressive—are adopted in this paper. The VA+GMWB is assumed to be invested in a moderate aggressive allocation throughout the study. The detailed asset allocations for the four model portfolios are shown in table 2A.

Table 2A. Asset Allocation Portfolios

Asset Classes	Benchmark	Conservative	Mod. Con.	Moderate	Mod. Aggr.
U.S. Large Cap Stocks	Russell 1000	10%	20%	30%	35%
U.S. Mid Cap Stocks	Russell MidCap	5%	10%	10%	15%
U.S. Small Cap Stocks	Russell 2000	0%	0%	5%	5%
International Stocks	MSCI EAFE	5%	10%	15%	25%
U.S. Aggregate Bonds	LB Aggr. Bond	40%	35%	25%	15%
Short-term Bonds	LB 1-5 Yr Gvt/Credit	25%	15%	10%	5%
Cash	CG U.S. 3 Mo Tbill	15%	10%	5%	0%

The conservative model portfolio has 20% equity and 80% fixed income (20/80). The moderate conservative model portfolio has 40% equity and 60% fixed income (40/60). The moderate model portfolio has 60% equity and 40% fixed income (60/40). Finally, the moderate aggressive allocation has 80% equity and 20% fixed income (80/20).

Table 2B shows the historical returns and standard deviations during1979-2006 and Ibbotson's forward-looking returns and standard deviations for the seven asset classes. Table 2C shows the historical correlation matrix (1979—2006) for the seven asset classes. The beginning year

1979 was chosen for the empirical study, simply because of the availability of data.[4] Table 2D shows the fee structure assumed in this study. For mutual fund portfolios, the total fees of 2% include 1% management fee and 1% advisor fee. For the VA+GMWB, the total fees of 3% include 1% underlying fund management fee, 1% advisor fee, 0.4% VA M&E fee, and 0.6% GMWB rider fee. The GMWB rider fee is based on the benefit base, while the other fees are based on the contract value.

The historical standard deviations shown in table 2B are calculated from annual returns instead of monthly returns, and Ibbotson forecasted standard deviations are estimated from annual returns. It is known that serial correlations among time-series returns tend to lower the standard deviation of returns; in particular, monthly cash returns are highly serial correlated. Using monthly returns, cash standard deviation is only 0.96%, while annual returns give a cash standard deviation of 3.34%. Also, the Monte Carlo simulations are conducted annually, therefore, it is consistent to use annual instead of monthly returns to estimate standard deviations for asset classes.

Table 2B. The historical returns and standard deviations (1979-2006), and Ibbotson's forecasted returns and forecasted standard deviations for the seven asset classes.

	1979--2006 Arithematic Average Return	1979--2006 Geometric Average Return	1979--2006 STD	Ibbotson E[R]*	Ibbotson STD
U.S. Large Cap Stocks	14.56%	13.46%	15.61%	11.03%	19.49%
U.S. Mid Cap Stocks	16.30%	15.31%	15.09%	12.61%	22.28%
U.S. Small Cap Stocks	14.83%	13.33%	18.54%	14.73%	28.74%
International Stocks	13.86%	11.90%	21.67%	11.39%	24.62%
U.S. Aggregate Bonds	9.07%	8.83%	7.47%	5.24%	7.05%
Short-term Bonds	8.25%	8.14%	5.04%	4.10%	4.21%
Cash	6.26%	6.21%	3.34%	3.48%	2.97%

* Ibbotson forward-looking annual expected return (arithmetic).

[4] We believe that our forward-looking return estimates are more realistic returns for today's investors than historical returns from 1979 to 2006.

Table 2C. Historical correlation matrix (1979—2006) for the seven asset classes

	Large Cap	Mid Cap	Small Cap	International	Aggr. Bonds	S/T Bonds	Cash
Large Cap	100%	95%	83%	57%	24%	17%	1%
Mid Cap	95%	100%	93%	55%	23%	17%	-1%
Small Cap	83%	93%	100%	52%	14%	8%	-2%
International	57%	55%	52%	100%	16%	13%	-5%
Aggr. Bonds	24%	23%	14%	16%	100%	96%	11%
S/T Bonds	17%	17%	8%	13%	96%	100%	23%
Cash	1%	-1%	-2%	-5%	11%	23%	100%

Table 2D. Assumed Fee Structure for Traditional Mutual Fund Portfolio and VA+GMWB

Fees Catogory[5]	Mutual Fund Portfolio	VA+GMWB
Fund Management Fee	1%	1 %
Advisor Fee*	1%	1%
VA M&E Fee	N/A	0.4%
GMWB Rider Fee	N/A	0.6%
Total Fees	2%	3%

* Advisor fee is the service fee charged by a financial advisor. We assume that both the traditional portfolio and the VA+GMWB are serviced through a financial advisor.

[5] The fees on both variable annuities and traditional mutual funds can vary greatly from product to product.

VA+GMWB Modeling

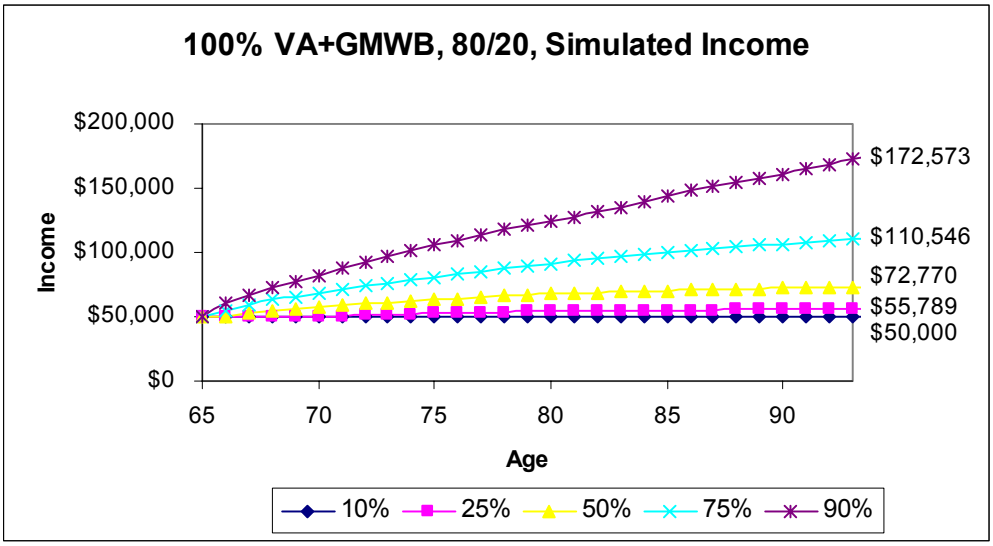
We followed Milevsky (2006) in modeling the dynamics of the traditional mutual fund portfolio value and the VA+GMWB contract value; the detailed formulas are shown in Appendix A. In each period, we generate seven random numbers for the seven asset classes, and we then calculate the portfolio return over the period. The retirement income is withdrawn at the beginning of the period, and meanwhile the fees are paid. This process will be repeated for 28 years for the empirical study or 30 years in the forward looking study; this completes one simulation path. The process is repeated 5,000 times to complete the entire simulation.

Simulated Guaranteed Income for VA+GMWB

Chart 2 shows the Monte Carlo simulated guaranteed income for the VA+GMWB assuming an initial investment of $1 million at age 65. The withdrawal rate is set at 5% of the benefits base. The asset allocation in the VA+GMWB account is moderate aggressive (80/20). Ibbotson's forward-looking asset returns and standard deviations, which are shown in table 2B, are used in the simulations. In chart 2, the 50-percentile income curve indicates the average income from VA+GMWB, while the 10th percentile is the worst scenario, i.e. the income is at least $50,000 when the market has tumbled for an extended period of time. The average ending income after 28 years is $72,770. The historical income levels shown in chart 1A fall slightly above the 90th line in chart 2, much higher than the 50th percentile simulated income. The reason is that our forecasted future returns are much lower than the historical returns from 1979 to 2006 shown in table 2B.

(For readers not familiar with percentiles in Monte Carlo simulation, table 7D in Appendix VI section B shows the implied returns at the various percentiles for the VA+GMWB over a 28-year horizon).

Chart 2. Income for 100% VA+GMWB



Semi-Deviation Measurement of Income Return

In this section, we study three model portfolios (conservative, moderate conservative, and moderate). In each model portfolio, we replace a portion of the fixed income/cash of the allocation with a more aggressively allocated VA, which will leave the remaining mutual fund portfolio with a heavier equity allocation than the original mutual fund portfolio. We will then study the impact of replacing some percentages of fixed income with the VA+GMWB on the average total income returns and semi-deviation of income returns for each model portfolio. The asset allocation for the VA+GMWB will be fixed at 80/20 (moderate aggressive).

For example, moderate conservative portfolio has an asset allocation of 40/60 (equity/fixed income). If 15% of the fixed income is replaced with 15% VA+GMWB, the remaining model portfolio will have 40% equity and 45% fixed income, or effectively 47/53 in a scale of 100%. In other words, the remaining model portfolio becomes slightly more aggressive (40/60 → 47/53). The combined portfolio has an allocation of 40/45/15 (equity / fixed income / VA+GMWB).

Chart 3A-B show the traditional efficient frontier for the seven asset classes, along with the four model portfolios and some selected combined portfolios. Ibbotson's forward-looking asset returns and standard deviations were used to generate the efficient frontier. The gross returns and net returns for the model portfolios and combined portfolios are shown in chart 3A and chart 3B, respectively. The assumed fee structure is shown in table 2D. The combined portfolios are slightly more aggressive than their corresponding model portfolios. For example, the 40/45/15 has a higher risk than the moderate conservative portfolio (40/60), but a lower risk than the moderate portfolio (60/40). The pure U.S. aggregate bonds and the pure large-cap stocks are plotted for comparisons. The combined 20/60/20 portfolio, in which 20% bonds were replaced with 20% VA+GMWB, is only slightly more aggressive than the pure U.S. aggregate bonds.

Chart 3A. Traditional Efficient Frontier (Gross Returns)

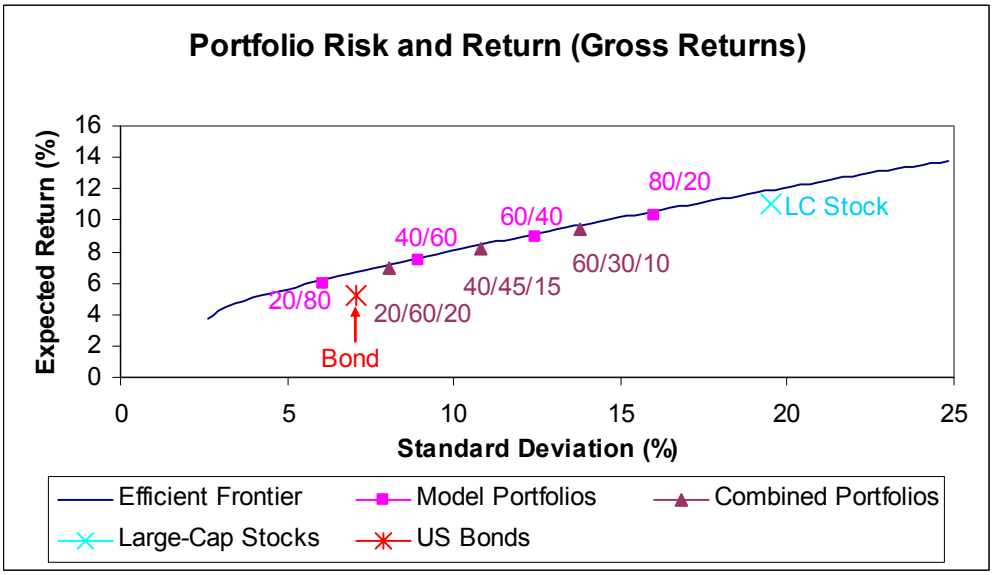
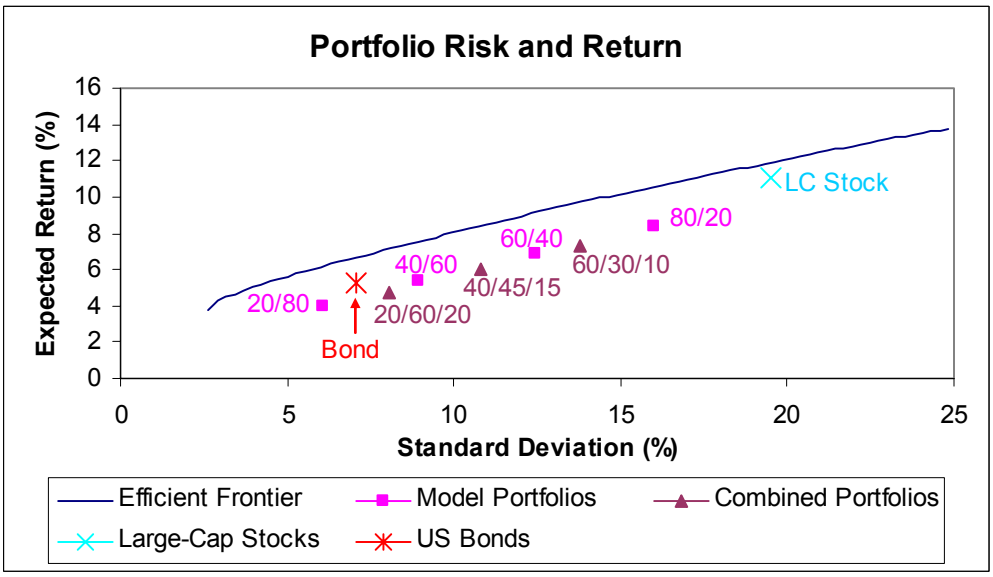


Chart 3B. Traditional Efficient Frontier (Net Returns)



* The assumed fee structure is shown in table 2D.

The combined portfolios have a higher underlying equity allocation than the original moderate conservative model portfolio. We will show next that the combined portfolios have lower income risk and higher total income returns than the moderate conservative portfolio, due to a slightly more aggressive allocation in the combined portfolios and the guaranteed withdrawals from the VA+GMWB.

We assume that the retirement income is from a fixed 5% withdrawal based on the beginning balance for each model portfolio. For the VA+GMWB portion, as we mentioned above, the income is the guaranteed 5% of the benefits base. The income return is defined as the percentage change in income for two consecutive years. Table 3 illustrates the calculation of income returns for the moderate conservative model portfolio as well as the combined portfolio (40/45/15) in which 15% VA+GMWB replaces bonds.

In table 3, at the beginning of 1979, a $1 million initial investment was made in a hypothetical moderate conservative portfolio as well as in a combined 40/45/15 portfolio. Columns 2-4 show the portfolio principals, incomes, and income returns for each year for the moderate conservative portfolio. Columns 5-7 show the portfolio principals, incomes, and income returns for each year for the combined 40/45/15 portfolio. At the end of 2006, the average income return, total income, and ending assets are 3.53%, $2,891,950, and $2,349,607 for the moderate conservative portfolio, respectively. The average income return, total income, and ending assets are 4.03%, $3,146,454, and $2,522,656 for the combined 40/45/15 portfolio, respectively. It can be seen that all three numbers (average income return, total incomes, and ending assets) are higher in the combined 40/45/15 portfolio.

Table 3. Historical Illustration of Income Return for the Moderate Conservative Portfolio (40/60), and the Combined Portfolio (40/45/15) from 1979 to 2006.

	40/60 Principal	40/60 Income	40/60 Inc. Ret.	40/45/15 Principal	40/45/15 Income	40/45/15 Inc. Ret.
	$1,000,000	$50,000		$1,000,000	$50,000	
1979	$1,036,555	$51,828	3.66%	$1,056,459	$52,823	5.65%
1980	$1,116,459	$55,823	7.71%	$1,169,723	$58,486	10.72%
1981	$1,087,921	$54,396	-2.56%	$1,127,031	$57,095	-2.38%
1982	$1,244,296	$62,215	14.37%	$1,262,154	$63,135	10.58%
1983	$1,329,097	$66,455	6.82%	$1,368,044	$68,402	8.34%
1984	$1,367,511	$68,376	2.89%	$1,388,455	$69,608	1.76%
1985	$1,613,130	$80,656	17.96%	$1,662,188	$83,109	19.40%
1986	$1,809,476	$90,474	12.17%	$1,896,227	$94,811	14.08%
1987	$1,777,671	$88,884	-1.76%	$1,873,059	$93,771	-1.10%
1988	$1,868,863	$93,443	5.13%	$1,996,399	$99,820	6.45%
1989	$2,050,512	$102,526	9.72%	$2,205,824	$110,291	10.49%
1990	$1,930,065	$96,503	-5.87%	$2,015,588	$103,718	-5.96%
1991	$2,166,473	$108,324	12.25%	$2,285,183	$114,696	10.58%
1992	$2,145,312	$107,266	-0.98%	$2,250,690	$113,574	-0.98%
1993	$2,232,547	$111,627	4.07%	$2,360,264	$118,013	3.91%
1994	$2,077,801	$103,890	-6.93%	$2,207,244	$111,644	-5.40%
1995	$2,351,531	$117,577	13.17%	$2,517,420	$125,871	12.74%
1996	$2,401,903	$120,095	2.14%	$2,605,132	$130,257	3.48%
1997	$2,570,673	$128,534	7.03%	$2,820,835	$141,042	8.28%
1998	$2,714,035	$135,702	5.58%	$3,011,045	$150,552	6.74%
1999	$2,764,837	$138,242	1.87%	$3,147,157	$157,358	4.52%
2000	$2,675,857	$133,793	-3.22%	$2,969,773	$151,546	-3.69%
2001	$2,479,542	$123,977	-7.34%	$2,667,569	$140,708	-7.15%
2002	$2,248,858	$112,443	-9.30%	$2,318,020	$127,888	-9.11%
2003	$2,434,423	$121,721	8.25%	$2,570,610	$138,038	7.94%
2004	$2,458,264	$122,913	0.98%	$2,614,704	$139,833	1.30%
2005	$2,412,125	$120,606	-1.88%	$2,572,595	$138,280	-1.11%
2006	$2,473,271	$123,664	2.53%	$2,664,741	$142,085	2.75%
	$2,349,607			$2,522,656		
		Total:	Average:		Total:	Average:
		$2,891,950	3.52%		$3,146,454	4.03%

The average loss income return is defined as the average of negative income returns and the semi-deviation is defined as the standard deviation of negative income returns. By design, the VA+GMWB has zero downside risk for the income returns due to the guaranteed minimum incomes. Therefore, the income risk only comes from the non-VA portion of the portfolios. To continue with the above-mentioned moderate conservative portfolio and the combined 40/45/15 portfolio shown in table 3, the average loss income return and semi-deviation are 4.43% and 2.98% for the moderate conservative portfolio, respectively. The average loss income return and semi-deviation are 4.1% and 2.97% for the combined 40/45/15 portfolio, respectively. One can see that the average loss income return is lower in the 40/45/15 portfolio and semi-deviations are almost same for the 40/60 and 40/45/15 portfolios.

The empirical data analysis shown in table 3 suggests that VA+GMWB will help improve the overall retirement income levels without increasing income risk levels, which is our hypothesis. However, one could argue that the history might not repeat, and such a strong market performance may not sustain long in the future. To test this hypothesis, Monte Carlo simulations with Ibbotson forward-looking returns and standard deviations are conducted for three model portfolios and their corresponding combined portfolios, and the simulated results along with empirical results are shown in tables 4–6.

The empirical results using historical returns from 1979-2006 for the conservative portfolio (20/80), and two combined portfolios (20/60/20 and 20/40/40) are shown in table 4A. All five measurements are attractive for the VA+GMWB combined portfolios, i.e., average total income return, total incomes, and ending assets are higher, while the average loss income return and loss semi-deviation are lower for the combined portfolios. The higher the VA+GMWB replacement ratio, the more attractive are the five measurements. Monte Carlo simulations with forward-looking expectations shown in table 4B largely support the above-mentioned hypothesis in all percentiles, except that the ending assets are lower for the combined portfolios for the 50, 25, and 10th percentiles. The reason is that a more aggressive portfolio suffers more, thus the ending value is less when the market has performed worse than average for an extended period of time.

To help understand the 90, 75, 50, 25, or 10th percentile concepts, Appendix B presents the implied portfolio returns for each percentile for all model portfolios and the corresponding combined portfolios.

Table 4A. Conservative Model Portfolio and Combined Portfolios from 1979 to 2006

	lossAvg	lossPeriodsIncome	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	-2.35%	$104,895	2.04%	2.21%	$2,386,967	$1,689,651
80%TA/20%VA	-2.31%	$123,117	1.49%	3.04%	$2,709,569	$1,891,567
60%TA/40%VA	-1.90%	$138,918	1.30%	3.74%	$3,030,080	$2,090,460

* lossAvg is the average of negative income returns; lossPeriodsIncome is the average income in the periods that have negative income returns; avgReturn is the average of both positive and negative income returns; totalWithdrawal is the total income amounts over the entire 28 years; endAsset is the ending assets of the portfolios.

Table 4B. Monte Carlo Simulations for Conservative Model Portfolio and Combined Portfolios

	percentiles	lossAvg	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	90%	-3.92%	2.58%	0.16%	$1,509,763	$955,892
Conservative	75%	-4.42%	2.95%	-0.41%	$1,349,058	$780,691
	50%	-4.96%	3.35%	-1.00%	$1,197,459	$630,198
	25%	-5.52%	3.78%	-1.60%	$1,075,338	$504,900
	10%	-6.05%	4.18%	-2.09%	$973,356	$423,807
80%TA/20%VA	90%	-2.77%	1.79%	1.31%	$1,895,864	$1,275,653
	75%	-3.14%	2.07%	0.43%	$1,607,263	$872,269
	50%	-3.55%	2.39%	-0.34%	$1,363,019	$581,920
	25%	-3.98%	2.72%	-0.94%	$1,199,025	$433,055
	10%	-4.38%	3.02%	-1.36%	$1,096,892	$352,067
60%TA/40%VA	90%	-1.98%	1.27%	2.28%	$2,330,908	$1,647,896
	75%	-2.30%	1.50%	1.15%	$1,882,060	$997,513
	50%	-2.63%	1.76%	0.14%	$1,523,754	$505,180
	25%	-2.95%	2.02%	-0.48%	$1,307,201	$339,173
	10%	-3.26%	2.27%	-0.86%	$1,200,195	$268,830

*Ending assets are the total of VA+GMWB contract value and traditional portfolio value for the combined portfolios.

The empirical results for the moderate conservative portfolio (40/60), and two combined portfolios (40/45/15 and 40/25/35) are shown in table 5A. The 40/45/15 portfolio was discussed before. The average total income return, total incomes, and ending assets are all higher in the combined portfolios. The average loss returns are lower in the combined 40/45/15 and 40/25/35 portfolios. The semi-deviation is slightly higher in the 40/25/35 combined portfolio because of the significant losses from 2000 to 2002, during which the stock market crashed while the bond market performed well, so that the semi-deviation is slightly higher for 40/25/35. Note that in table 5A, the average income during the income-loss periods is higher for 40/25/35 although its semi-deviation is higher.

Monte Carlo simulations on forward-looking market expectations shown in table 5B once again largely support the above-mentioned hypothesis in all percentiles, except that the ending assets are lower for the combined portfolios for the 50, 25, and 10th percentiles (ending value in a more aggressive portfolio suffers more when the market performed worse than average). The results are basically similar to the conservative portfolio shown in table 4B.

In contrast to empirical results, Monte Carlo simulations show that the semi-deviation is lower for the combined 40/25/35 portfolio. The reason is that Monte Carlo-simulated semi-deviation is the median semi-deviation over 5,000 possible scenarios, whereas the empirical semi-deviation is over just one historical scenario which may suffer from random fluctuations and extreme events.

Table 5A. Moderate Conservative Portfolio and Combined Portfolios from 1979 to 2006.

	lossAvg	lossPeriodsIncome	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	-4.43%	$120,614	2.98%	3.52%	$2,891,950	$2,349,607
85%TA/15%VA	-4.10%	$131,368	2.97%	4.03%	$3,146,454	$2,522,656
65%TA/35%VA	-3.87%	$145,755	3.27%	4.67%	$3,491,954	$2,762,954

Table 5B. Monte Carlo Simulations for Moderate Conservative Model Portfolio and Combined Portfolios

	percentiles	lossAvg	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	90%	-4.86%	3.27%	1.80%	$1,999,675	$1,628,291
Mod. Con.	75%	-5.54%	3.75%	0.98%	$1,692,730	$1,216,447
	50%	-6.28%	4.32%	0.10%	$1,419,155	$893,563
	25%	-7.11%	4.89%	-0.77%	$1,195,906	$652,495
	10%	-7.82%	5.49%	-1.49%	$1,039,492	$502,454
85%TA/15%VA	90%	-4.22%	2.79%	2.47%	$2,337,158	$1,943,362
	75%	-4.79%	3.22%	1.49%	$1,908,742	$1,322,344
	50%	-5.45%	3.72%	0.48%	$1,540,280	$865,718
	25%	-6.13%	4.25%	-0.39%	$1,281,492	$590,388
	10%	-6.78%	4.75%	-1.04%	$1,120,878	$436,589
65%TA/35%VA	90%	-3.66%	2.45%	3.35%	$2,857,450	$2,461,199
	75%	-4.14%	2.82%	2.12%	$2,216,999	$1,486,725
	50%	-4.72%	3.27%	0.94%	$1,705,451	$822,645
	25%	-5.28%	3.74%	0.02%	$1,390,871	$493,038
	10%	-5.88%	4.21%	-0.62%	$1,216,607	$342,876

The empirical results for the moderate portfolio (60/40) and two combined portfolios (60/30/10 and 60/15/25) are shown in table 6A. Monte Carlo simulation results are shown in table 6B. All observations in both empirical results and Monte Carlo simulations are similar to the moderate conservative portfolio (40/60) and the combined 40/45/15 and 40/25/35 portfolios.

Table 6A. Moderate Portfolio and Combined Portfolios from 1979 to 2006

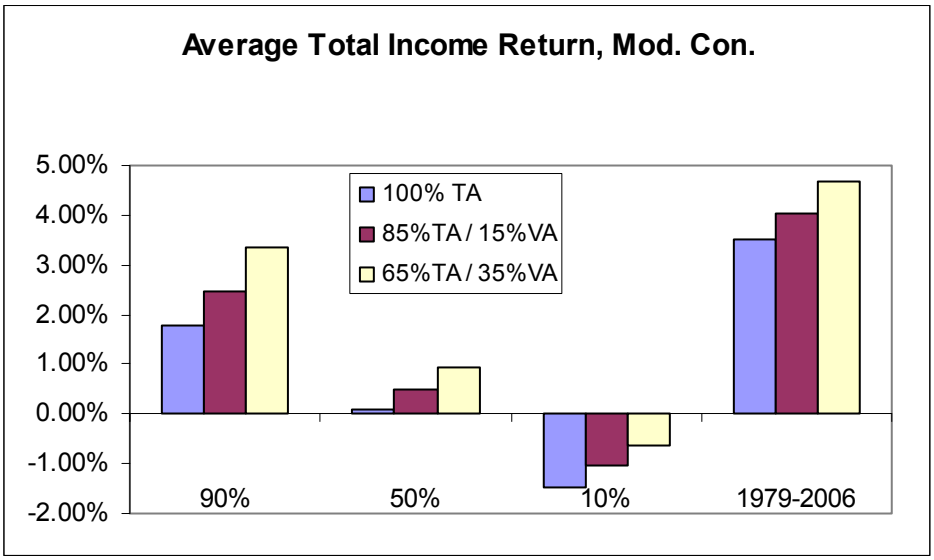
	lossAvg	lossPeriodsIncome	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	-6.29%	$141,218	4.81%	4.71%	$3,442,243	$3,095,772
90%TA/10%VA	-6.15%	$148,690	4.97%	5.02%	$3,626,326	$3,236,399
75%TA/25%VA	-6.28%	$144,305	5.24%	5.49%	$3,911,492	$3,460,556

Table 6B. Monte Carlo Simulations for Moderate Model Portfolio and Combined Portfolios

	percentiles	lossAvg	lossSTD	avgReturn	totalWithdrawal	endAsset
100%TA	90%	-6.10%	4.14%	3.60%	$2,744,220	$2,787,952
Moderate	75%	-7.03%	4.80%	2.48%	$2,152,300	$1,876,894
	50%	-8.04%	5.54%	1.24%	$1,672,398	$1,231,926
	25%	-9.12%	6.33%	0.05%	$1,313,599	$805,615
	10%	-10.16%	7.09%	-0.92%	$1,083,316	$561,032
90%TA/10%VA	90%	-5.82%	3.94%	4.02%	$3,026,717	$3,127,386
	75%	-6.67%	4.58%	2.76%	$2,322,990	$1,983,961
	50%	-7.60%	5.27%	1.46%	$1,758,553	$1,227,080
	25%	-8.57%	6.02%	0.24%	$1,369,233	$759,846
	10%	-9.51%	6.77%	-0.73%	$1,130,828	$513,107
75%TA/25%VA	90%	-5.57%	3.77%	4.71%	$3,539,081	$3,779,150
	75%	-6.32%	4.36%	3.24%	$2,597,461	$2,202,152
	50%	-7.17%	5.03%	1.78%	$1,895,225	$1,221,942
	25%	-8.06%	5.79%	0.51%	$1,445,823	$686,128
	10%	-9.04%	6.52%	-0.49%	$1,194,085	$426,319

Chart 4 A-E shows the average income return, average loss income return, semi-deviation, total income, and ending assets for the moderate conservative (40/60) portfolio and combined portfolios (40/45/15 and 40/25/35), which were discussed previously. The empirical period is from the beginning of 1979 to the end of 2006. The Monte Carlo simulations for the five parameters in the 90, 50 and 10th percentiles are plotted for comparison purposes and the period covered is 28-years.

Chart 4A. Average Total Income Returns for the 40/60, 40/45/15, and 40/25/35 Portfolios.



*90%, 50%, and 10% are the 90, 50, and 10th percentiles results for the Monte Carlo Simulations.

Chart 4B. Average Loss Income Returns for the 40/60, 40/45/15, and 40/25/35 Portfolios.

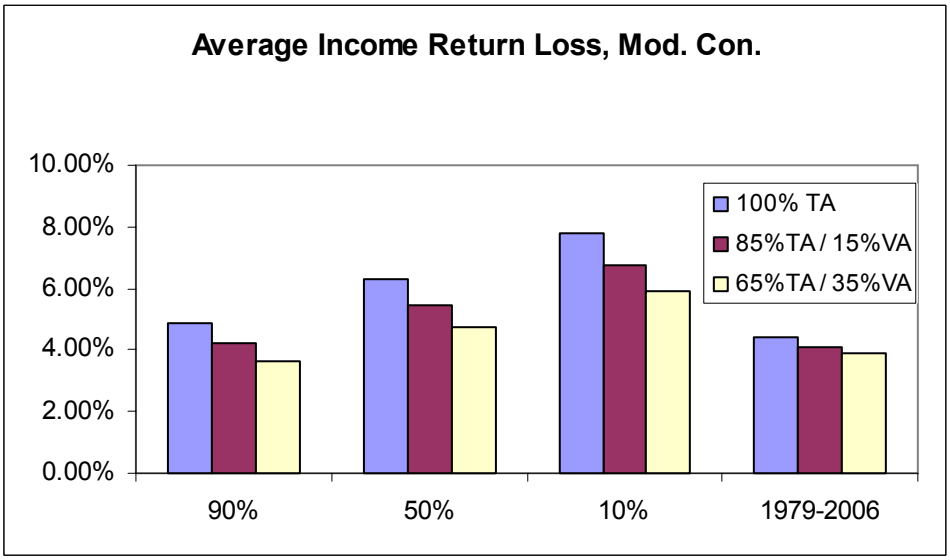
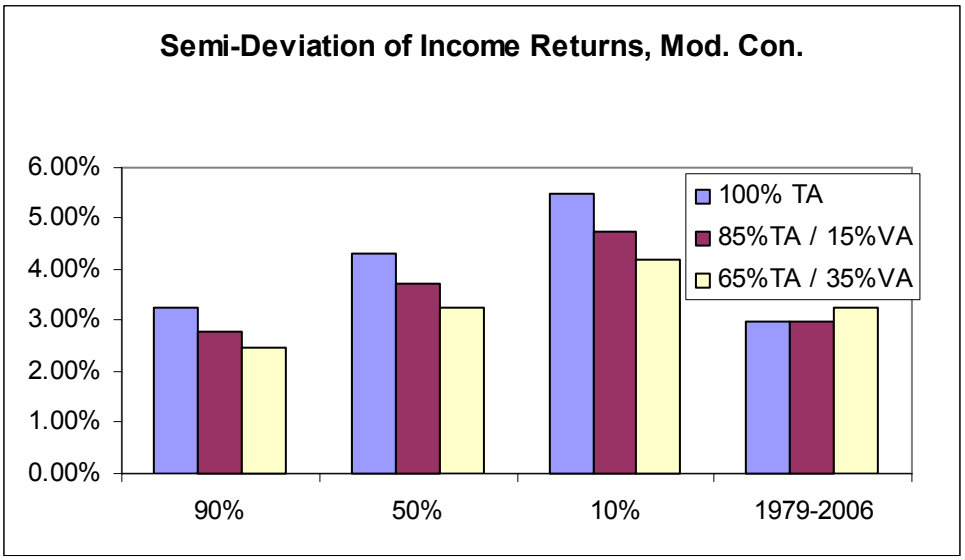


Chart 4C. Semi-Deviations of Income Returns for the 40/60, 40/45/15, and 40/25/35 Portfolios.



* Note that for historical analysis, the historical average income during the income-loss periods is $145,755 for 40/25/35 (higher than $120,614 for 40/60) although its semi-deviation is slightly higher.

Chart 4D. Total Incomes for the 40/60, 40/45/15, and 40/25/35 Portfolios.

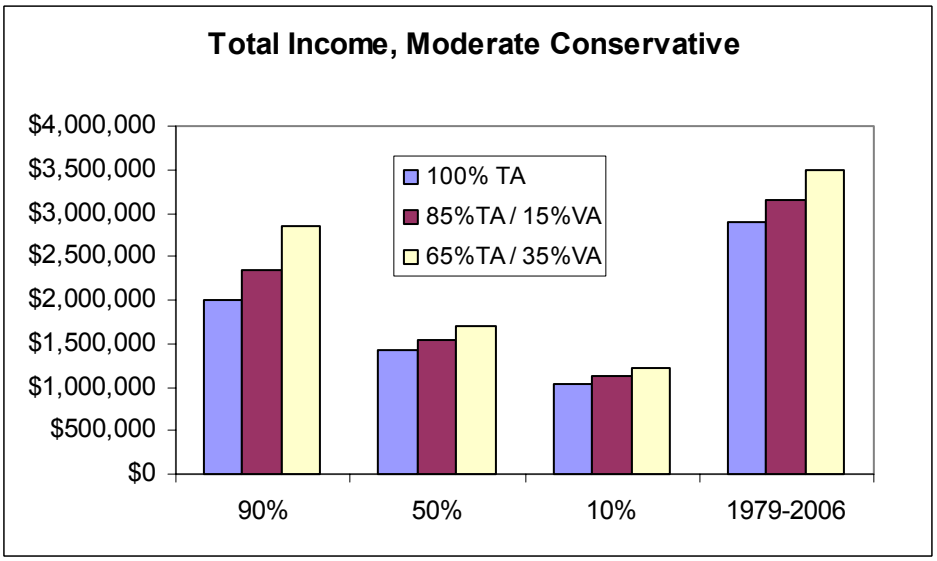
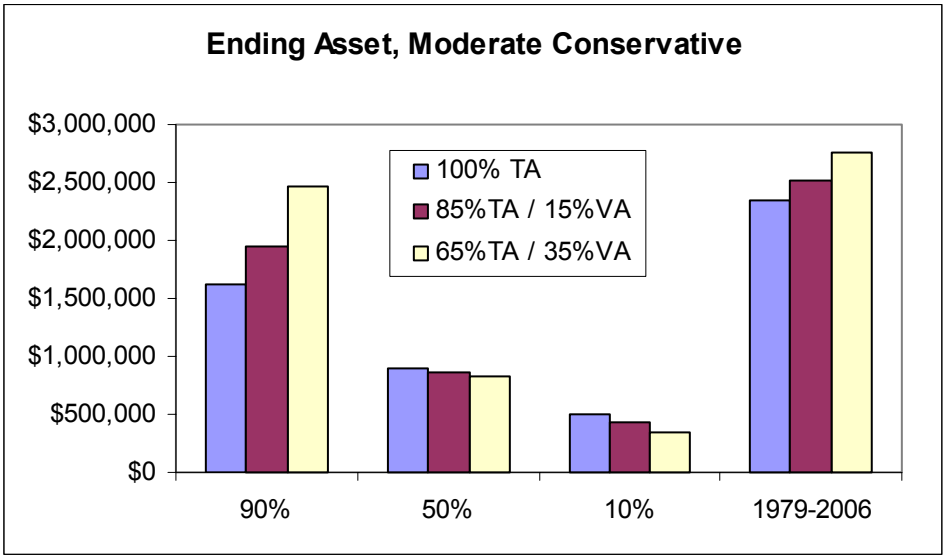


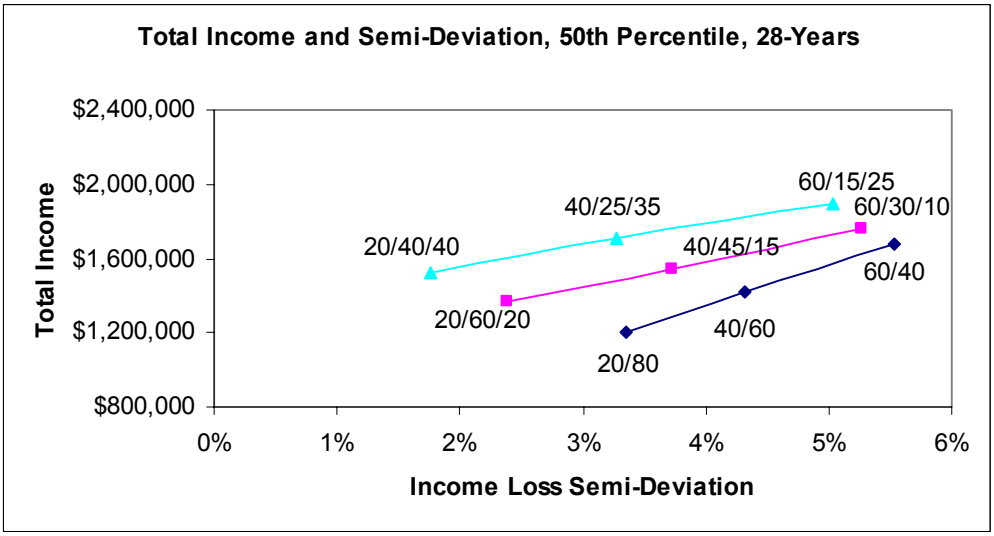
Chart 4E. Ending Assets for the 40/60, 40/45/15, and 40/25/35 Portfolios.



Proof of Hypothesis

The total income and semi-deviation for conservative, moderate conservative, moderate portfolios, and their corresponding combined portfolios are summarized in Chart 5. It plots the 50th percentile of total income and semi-deviation from the Monte Carlo simulations with forward-looking market expectations over the 28 years. In general, more aggressive model portfolios have higher total income and higher semi-deviation (thus higher income risk), however, additions of VA+GMWB will shift the income-risk curve in Chart 5 up and left. That is, the guaranteed income from VA+GMWB has no income risk which helps to lower the overall income risk, and the combined portfolios have more equity allocation which contributes to an increase in the total income.

Chart 5. Monte Carlo Simulations for Total Income and Semi-Deviation



*The assumed fee structure is shown in table 2D.

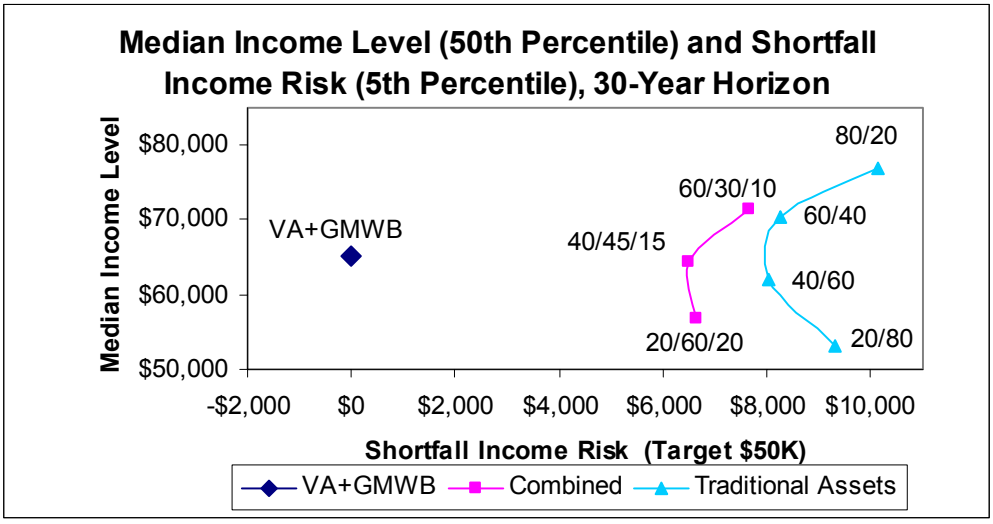
Average Sustainable Income Level and Shortfall Income Risk

In this section, we introduce another framework, i.e. average sustainable income level vs. shortfall income risk. It differs from the previously discussed semi-deviation analysis in that the income withdrawal in this framework is in constant or slightly increasing dollar amounts, instead of a fixed 5% based on the portfolio value in each year. The annual fixed 5% of withdrawal rate based on the portfolio value is assumed in the semi-deviation framework.

The shortfall income risk is defined as the shortage of income compared to a target income of $50,000 at the 5th percentile for a given investment horizon, e.g. 30 years. The 5th percentile is chosen to represent the "worst" market return scenario for the portfolio performance. The average sustainable income level is the annual income that can sustain for 30 years at the 50th percentile, and more precisely, it is the median sustainable income level.

Chart 6 shows the average or median sustainable income level and shortfall risk for a 30-year horizon for three sets of portfolios: 1) stand-alone VA products with these guarantees (VA+GMWB); 2) stand-alone traditional non-annuity products (such as mutual funds); and 3) a combination of VA products and non-annuity products. Expected returns and standard deviations for the seven asset classes used in the simulation analysis are forecasted by Ibbotson Associates and shown in table 2B.

Chart 6. Median Income Level and Shortfall Risk for 30-Year Horizon



For stand-alone VA+GMWB, the shortfall income is $0 because the income is at least 5% of the initial investment of $1,000,000 (=$50,000). For the conservative portfolio, at the 5th percentile, the income is only $40,700 for a 30-year horizon, thus the shortfall income is $9,300 (=$50,000-$40,700). However, in the combined 20/60/20 (20% bonds were replaced by 20% VA+GMWB in the conservative portfolio) the shortfall income declined to $6,650.

The average or median sustainable income level for stand-alone VA+GMWB is simply the average income payout at the 50th percentile for the 30-year horizon. For stand-alone traditional asset portfolios or combined portfolios, the average income levels are calculated by binary searches, and in addition, two conditions are satisfied for apples-to-apples comparisons: (a) the ending total portfolio value is equal to the ending contract value of the stand-alone VA+GMWB at the 50th percentile and (b) the income stream shape is the same as the stand-alone VA+GMWB at the 50th percentile shown in Chart 2.

The amount of income withdrawn from VA+GMWB is dynamically determined by the benefits base, and after 30 years, the VA+GMWB may still have positive remaining contract value. The above-mentioned condition (a) considers the impact on positive remaining contract value. On the other hand, different withdrawal patterns or income curves result in different ending portfolio values, and condition (b) takes it into account. Therefore, the same average ending assets and the same income stream shape allow unbiased average income levels to be calculated for the traditional and combined portfolios. For the conservative 20/80 portfolio, the average sustainable income level is $53,068, while for the combined 20/60/20 portfolio, the average sustainable income level increased to $56,766.

Chart 6 clearly shows that all combined portfolios have higher average income levels than stand-alone traditional mutual fund portfolios, and for conservative, moderate conservative, and moderate portfolios, additions of VA+GMWB even lead to a reduction in shortfall income risk due to the guaranteed income feature in the VA+GMWB. In other words, adding VA+GMWB to the conservative, moderate conservative, and moderate model portfolios enhances average income and reduces shortfall risk for investors needing roughly 5% from their portfolios to sustain retirement income for 30 years or more[6,7].

[6] Investors needing much higher or much lower percentage of withdrawal from their portfolio to sustain retirement income would still benefit from including VA+GMWB; however the benefit might not be as much as those illustrated in this paper.

[7] These findings suggest that adding VA+GMWB to the retirement portfolio can be beneficial. We intend to address how much an investor should put into VA+GMWB in a retirement portfolio in a separate analysis.

V. Conclusions

We applied the semi-deviation measurements on income returns for a series of simulation analyses across the three scenarios: 1) stand-alone VA products with these guarantees; 2) stand-alone traditional non-annuity products (such as mutual funds); and 3) a combination of VA products and non-annuity products in a portfolio context. In the combined portfolio, a portion of the fixed income /cash is replaced with a more aggressively allocated VA, which will leave the remaining mutual fund portfolio with a heavier equity allocation than the original mutual fund portfolio.

The combined portfolios have a slightly more aggressive allocation than the original moderate conservative model portfolio. However, we have shown that the combined portfolios have lower income risk and higher total income returns than the corresponding portfolios, due to the more aggressive allocation and the guaranteed withdrawals in the VA+GMWB.

Assuming a fixed percentage withdrawal rate (5%) on the model portfolios, more aggressive model portfolios provide higher total income with higher semi-deviation (thus higher income risk). However, both empirical results and Monte Carlo simulations show that the combined portfolios have lower average negative income return and semi-deviation and higher average total income return and total income withdrawals. The addition of VA+GMWB helps to increase total income while reducing income risk. That is, the guaranteed income from VA+GMWB has no income risk which helps to lower the overall income risk, and the combined portfolios have more equity allocation which contributes to increase total income. The tradeoff is that the ending portfolio assets are lower in the combined portfolios when the market performed worse than average for extended periods of time. However, if the market performed better than average, the ending portfolio assets are likely to be higher in the combined portfolios.

In an alternative framework, we analyzed the shortfall risk and median sustainable income level for the three model portfolios and the combined portfolios over the 30-year horizon. We found that all combined portfolios have higher average income levels and lower shortfall income risk than stand-alone traditional mutual fund portfolios. In other words, adding VA+GMWB to the conservative and moderate conservative model portfolios enhances average sustainable income while reducing shortfall income risk.

While presenting the benefit of a VA+GMWB from the results, we need to keep in mind a few caveats of this study. First, the study assumes the investor has a retirement income period of 25 years or longer (28 in the income risk analysis and 30 years in the income shortfall risk analysis, to be precise), which we believe is the typical planning horizon for retirees today and in the future, who have a significant probability of living pass age 85 and 90. This probability is particular high for married couples. For an investor with a shorter horizon, a VA+GMWB may not be as beneficial. On the other hand, for investors with even longer horizons (e.g., 30+ years), the benefits will be greater than those presented in this study. Second, the study does not consider the default risk in VA contracts; we believe this risk is very small. Third, we focus on investors needing roughly 5% from one's portfolios; the benefit of GMWB will be lower for investors needing a much lower percentage of withdrawal (i.e., ultra high net worth investors). Lastly, for investors with existing market risk and retirement income risk coverage, the benefit will be smaller. An example would be investors with significant defined benefit pension income in retirement.

Overall, we believe VA+GMWB offers protection both in terms of market downturns and more importantly retirement income risk. For the typical investor in or near retirement, there is a good amount of value by investing a portion of his or her investment asset into VA+GMWB, which provides a guaranteed income level through retirement. This can potentially increase the amount of income generated from the entire portfolio (especially during poor market performance periods), and reduce the amount of income risk. Similar to payout annuities, VA+GMWB offers a secured lifetime income that is not available through traditional investment products. For the typical retiree, it is beneficial to look into investing a portion of their assets into VA+GMWB.

VI. Appendix

A. VA+GMWB Modeling

The dynamics of the VA+GMWB contract value is (Milevsky 2006):

$$dV_t = (\mu - \varepsilon)V_t + \sigma V_t dB_t - fM_t dt - gM_t dt \qquad (1)$$

$$M_t = \max(V_s, 0 \le s \le t)$$

$$Income_t = gM_t$$

Where V_t and M_t are the VA account or contract value and benefits base at time t, respectively. M_t is the maximum of the contract value from initial purchase to current period, i.e. the high-water mark of the contract value. B_t denotes a standard Brownian motion with mean zero and variance t. σ is the volatility of the VA account. The constant g is the guaranteed withdrawal rate (e.g. 5% of the initial deposit). The constant ε is the management fees for the VA account including underlying fund fees. The constant f is the GMWB rider fee (e.g. 0.6%). The spending or income is stepped up each time the contract or account value reaches a new maximum.

The g value can be age-dependent (e.g. 5% if income starts at age 60, and 5.5% if income starts at age 67.)

In contrast, for a traditional mutual fund portfolio, assuming a constant withdrawal rate (e.g. 5%) on the beginning balance of the portfolio in each period, the dynamics of the portfolio value is:

$$dV_t = (\mu - \varepsilon)V_t + \sigma V_t dB_t - gV_t dt \qquad (2)$$

$$Income_t = gV_t$$

The constant ε is the mutual fund management fees. σ is the volatility of the mutual fund portfolio. The constant g is the withdrawal rate on the beginning balance of the mutual fund portfolio (5%) in each period.

B. Implied Returns

To give an estimate on how an average portfolio (at 50th percentile) or the worst scenario portfolio (at 10th percentile) performed over a 28-year horizon, we performed Monte Carlo simulations on the four model portfolios and some combined portfolios to get a wealth distribution at the end of 28 years. Assuming no intermediate cash flows and initial $1 million investments, we calculate the implied returns for each percentile based on the wealth distribution at the end of 28 years.

For example, for the moderate aggressive model portfolio at the 10th percentile, the ending portfolio is $2,610,016, and the implied return over the 28-year period is:

$(2.61)^{\wedge}(1/28) - 1 = 3.49\%$

Table 7A-D shows the implied returns for the four model portfolios and the corresponding combined portfolios. The assumed fee structure is shown in table 2D.

Table 7A. The implied returns for the conservative model portfolio and the corresponding combined portfolios over a 28-year horizon

Percentile	Conservative	20/60/20	20/40/40
90%	5.31%	6.47%	7.64%
75%	4.56%	5.45%	6.38%
50%	3.79%	4.46%	5.04%
25%	2.96%	3.42%	3.72%
10%	2.33%	2.58%	2.63%

Table 7B. The implied returns for the moderate conservative model portfolio and the corresponding combined portfolios over a 28-year horizon

Percentile	Moderate Conservative	40/45/15	40/25/35
90%	7.24%	8.13%	9.32%
75%	6.15%	6.80%	7.66%
50%	5.02%	5.46%	6.00%
25%	3.87%	4.09%	4.31%
10%	2.94%	2.98%	2.87%

Table 7C. The implied returns for the moderate model portfolio and the corresponding combined portfolios over a 28-year horizon

Percentile	Moderate	60/30/10	60/15/25
90%	9.26%	9.84%	10.69%
75%	7.73%	8.16%	8.74%
50%	6.18%	6.44%	6.81%
25%	4.63%	4.71%	4.80%
10%	3.32%	3.26%	3.12%

Table 7D. The implied returns for the moderate aggressive model portfolio and VA+GMWB over a 28-year horizon

Percentile	Moderate Aggressive	VA+GMWB
90%	11.09%	10.09%
75%	9.15%	8.15%
50%	7.16%	6.16%
25%	5.17%	4.17%
10%	3.49%	2.49%

C. Glossary

Benefit Base: a high-water mark of the VA+GMWB contract value, which is used to calculate the withdrawal income and rider charge.

Implied Return at 50th Percentile: a fixed compound return required to accumulate the wealth equals a Monte Carlo simulated wealth at the 50th percentile for a given period.

Income Return: the change of income in percentage from year to year.

Income Risk: the downside volatility (or semi-deviation) of the change of income from year to year.

Income Shortfall Risk: the shortage of sustainable income to a target income over a given period (e.g. 30 years).

Semi-Deviation: standard deviation of negative income returns.

VA+GMWB: variable annuity with a guaranteed minimum withdrawal benefit.

VII. Reference

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